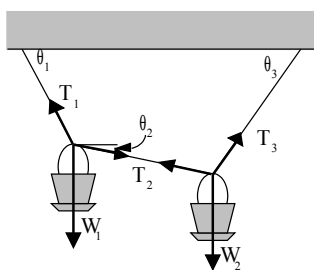


Chapter 5 Problem 71 †



Given

$$m_1 = 3.85 \text{ kg}$$

$$m_2 = 9.28 \text{ kg}$$

$$\theta_1 = 54.0^\circ$$

$$\theta_2 = 13.9^\circ$$

$$\theta_3 = 68.0^\circ$$

Solution

Find the tension in each of the wires.

The free body diagrams for the two plants are drawn on the picture. The coordinate system is chosen to have the x-axis in the horizontal and the y-axis in the vertical. Write out Newton's 2nd law for the first plant.

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{T}_1 + \vec{T}_2 + \vec{W}_1 = m_1\vec{a}$$

The plant is stationary so there is no acceleration. The equation in unit vector notation is

$$-T_1 \cos \theta_1 \hat{i} + T_1 \sin \theta_1 \hat{j} + T_2 \cos \theta_2 \hat{i} - T_2 \sin \theta_2 \hat{j} - m_1 g \hat{j} = 0$$

The x-component is then

$$-T_1 \cos \theta_1 + T_2 \cos \theta_2 = 0 \tag{1}$$

and the y-component is

$$T_1 \sin \theta_1 - T_2 \sin \theta_2 - m_1 g = 0 \tag{2}$$

Write out Newton's 2nd law for the second plant.

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{T}_2 + \vec{T}_3 + \vec{W}_2 = m_2\vec{a}$$

The plant is stationary so there is no acceleration. The equation in unit vector notation is

$$-T_2 \cos \theta_2 \hat{i} + T_2 \sin \theta_2 \hat{j} + T_3 \cos \theta_3 \hat{i} + T_3 \sin \theta_3 \hat{j} - m_2 g \hat{j} = 0$$

The x-component is then

$$-T_2 \cos \theta_2 + T_3 \cos \theta_3 = 0 \tag{3}$$

†Problem from Essential University Physics, Wolfson

and the y-component is

$$T_2 \sin \theta_2 + T_3 \sin \theta_3 - m_2 g = 0 \quad (4)$$

Take equation (1) and solve for T_1 .

$$T_1 = \frac{\cos \theta_2}{\cos \theta_1} T_2 \quad (5)$$

Substitute equation (5) into (2) and solve for T_2 .

$$\frac{\cos \theta_2}{\cos \theta_1} T_2 \sin \theta_1 - T_2 \sin \theta_2 - m_1 g = 0$$

$$T_2 (\cos \theta_2 \tan \theta_1 - \sin \theta_2) - m_1 g = 0$$

$$T_2 = \frac{m_1 g}{\cos \theta_2 \tan \theta_1 - \sin \theta_2} = \frac{(3.85 \text{ kg})(9.8 \text{ m/s}^2)}{\cos(13.9^\circ) \tan(54.0^\circ) - \sin(13.9^\circ)}$$

$$T_2 = 34.4 \text{ N}$$

Substitute this result into equation (5) and get

$$T_1 = \frac{\cos(13.9^\circ)}{\cos(54.0^\circ)} (34.4 \text{ N}) = 56.8 \text{ N}$$

Now take equation (3) and solve for T_3 .

$$T_3 = \frac{\cos \theta_2}{\cos \theta_3} T_2 = \frac{\cos(13.9^\circ)}{\cos(68.0^\circ)} (34.4 \text{ N})$$

$$T_3 = 89.1 \text{ N}$$