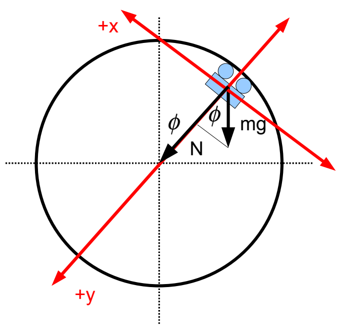
Chapter 5 Problem 59 [†]



Given

Solution

Find the angle at which the cart loses contact with the vertical circular track.

For the instant of time illustrated in the diagram, the angle between vertical and the cart's position is ϕ. A coordinate system is chosen such that the positive y direction is towards the center of the circle and the positive x direction is in the direction of motion of the cart. Using Newton's 2nd law we get

$$\Sigma\vec{F} = m\vec{a}$$

$$\vec{N} + \vec{W} = m\vec{a} \tag{1}$$

Now using the designated coordinate system

$$\vec{N} = N\hat{j}$$

$$\vec{W} = -mg\sin\phi\hat{i} + mg\cos\phi\hat{j}$$

Since the cart is experiencing centripetal acceleration up to the point that it leaves the track, the acceleration is towards the center of the circle and is, therefore,

$$\vec{a} = a\hat{j}$$

Substituting into equation 1 gives

$$N\hat{j} - mg\sin\phi\hat{i} + mg\cos\phi\hat{j} = ma\hat{j}$$

Looking at the equation in the y direction we have

$$N + mg\cos\phi = ma$$

Just as the cart loses contact with the track, the normal force is zero. Therefore,

$$mg\cos\phi = ma$$

Since centripetal acceleration is $a = v^2/r$, then

$$mg\cos\phi = m\frac{v^2}{r}$$

Solving for $\cos\phi$ gives

$$\cos\phi = \frac{mv^2}{mgr} = \frac{v^2}{rg}$$

[†]Problem from Essential University Physics, Wolfson