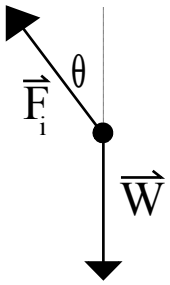
Chapter 5 Problem 40 [†]



Given
$m = 45\ kg$
$r = 5.0\ m$
$v = 6.3\ m/s$

Solution
a) Find the horizontal and vertical components of force exerted on the skate blades.

The free-body diagram is given above. Chose the x-coordinate to be to the right. Using Newton's 2^{nd} law

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{F_i} + \vec{W} = m\vec{a}$$

Since the skater is going around a circle at constant speed the acceleration must be centripetal acceleration. The direction of this acceleration is in the -x direction. Write out the equation in unit vector notation.

$$-F_i \sin\theta \hat{i} + F_i \cos\theta \hat{j} - mg\hat{j} = -m\frac{v^2}{r}\hat{i}$$

The x-component of this equation is

$$-F_i \sin\theta = -m\frac{v^2}{r} \tag{1}$$

and the y-component of this equation is

$$F_i \cos\theta - mg = 0 \tag{2}$$

In equation (1) the horizontal component of the force on the skate blades is $F_i \sin\theta$. Solving for this quantity gives

$$F_h = F_i \sin\theta = m\frac{v^2}{r} = (45\ kg)\frac{(6.3\ m/s)^2}{(5.0\ m)} = 357\ N$$

The vertical component of the force on the skate blades is equal to the normal force which is the quantity $F_i \cos\theta$. From equation (2) this is

$$F_v = F_i \cos\theta = mg = (45\ kg)(9.8\ m/s^2) = 441\ N$$

[†]Problem from Essential University Physics, Wolfson

b) Find the angle the skater is leaning without falling over.

The angle θ is the tangent of the opposite side (horizontal component) divided by the adjacent side (vertical component). This gives

$$\tan\theta = \frac{F_h}{F_v}$$

$$\theta = \tan^{-1}\left(\frac{F_h}{F_v}\right) = \tan^{-1}\left(\frac{357\ N}{441\ N}\right) = 39.0^\circ$$