Chapter 3 Problem 54 [†]

Given

$\vec{v}_0 = \left\{11\hat{i} + 14\hat{j}\right\}\ m/s$

$\vec{a} = \left\{-1.2\hat{i} + 0.26\hat{j}\right\}\ m/s^2$

Particle begins at the origin.

Solution

a) When does the particle cross the y axis?

The position vector of the particle is

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{r} = 0 + \left\{11\hat{i} + 14\hat{j}\right\} t + \tfrac{1}{2}\left\{-1.2\hat{i} + 0.26\hat{j}\right\} t^2$$

Regrouping gives

$$\vec{r} = \left\{(11t - 0.60t^2)\hat{i} + (14t + 0.13t^2)\hat{j}\right\}\ m$$

The particle crosses the y axis when the x component equals zero.

$$11t - 0.60t^2 = 0$$

$$(11 - 0.60t)t = 0$$

The solutions to this equation are $t = 0\ s$ and $t = 18.3\ s$. The first solution is the initial condition of the problem. The second solution is when it crosses back again. Therefore,

$$t = 18.3\ s$$

b) What is the y coordinate at this time?

Take the y component of the position vector and substitute in $t = 18.3\ s$.

$$y = 14(18.3) + 0.13(18.3)^2 = 300\ m$$

c) Find the speed and direction of the particle at this time.

The velocity at $t = 18.3\ s$ is

$$\vec{v} = \vec{v}_0 + \vec{a}t = \left\{11\hat{i} + 14\hat{j}\right\}\ m/s + \left\{-1.2\hat{i} + 0.26\hat{j}\right\}\ m/s^2(18.3\ s)$$

$$\vec{v} = \left\{-11.0\hat{i} + 18.8\hat{j}\right\}\ m/s$$

The magnitude of this vector is

$$v = \sqrt{(-11.0)^2 + (18.8)^2}\ m/s$$

$$v = 21.8\ m/s$$

The direction of this vector is in the second quadrant.

$$\theta = tan^{-1}\left(\frac{18.8}{-11.0}\right) = 120^\circ.$$

[†]Problem from Essential University Physics, Wolfson