Chapter 13 Problem 51 †

Given

$L = 8.0\ m$
$m_{man} = 75\ kg$
$\%\ change = -20\ \%$

Solution

Find the mass of the beam.

The frequency of the beam's oscillations change by 20%. Since frequency and angular frequency are related to each other by the formula, $\omega = 2\pi f$, the angular frequency also changes by 20%. The angular frequency of a torsional pendulum is

$$\omega = \sqrt{\frac{\kappa}{I}} \tag{1}$$

where k is the torsional constant and I is the moment of inertia. The moment of inertia before the men jump on the beam is

$$I_{before} = \tfrac{1}{12}ML^2$$

In this case we are assuming the beam acts like a uniform rod rotating about its center of gravity. Substituting this into equation 1 gives an angular frequency of

$$\omega_{before} = \sqrt{\frac{\kappa}{\frac{1}{12}ML^2}}$$

The moment of inertia after the men mount the beam is

$$I_{after} = \tfrac{1}{12}ML^2 + m_{\text{man}}\left(L/2\right)^2 + m_{\text{man}}\left(L/2\right)^2$$

The moment of inertia each man adds to the system is their mass times the distance from the pivot point squared. The distance the men are from the pivot point is $L/2$. Substituting this moment of inertia into equation 1 gives an angular frequency of

$$\omega_{after} = \sqrt{\frac{\kappa}{\frac{1}{12}ML^2 + m_{\text{man}}L^2/2}}$$

The torsional constant is related to the torque exerted on the beam by the cable. This does not change by adding the two men. Since ω_{after} is 20% less than ω_{before}, the ratio of the two is

$$\frac{\omega_{after}}{\omega_{before}} = \frac{\omega_{before}(1-0.20)}{\omega_{before}} = 0.80$$

Substituting in the formulas for ω_{after} and ω_{before}, gives

$$0.80 = \frac{\omega_{after}}{\omega_{before}} = \frac{\sqrt{\frac{\kappa}{\frac{1}{12}ML^2 + m_{\text{man}}L^2/2}}}{\sqrt{\frac{\kappa}{\frac{1}{12}ML^2}}}$$

$$0.80 = \sqrt{\frac{\frac{1}{12}ML^2}{\frac{1}{12}ML^2 + m_{\text{man}}L^2/2}} = \sqrt{\frac{\frac{1}{12}M}{\frac{1}{12}M + m_{\text{man}}/2}}$$

†Problem from Essential University Physics, Wolfson

Square both sides and solve for M.

$$0.64\left(\tfrac{1}{12}M + m_{\text{man}}/2\right) = \tfrac{1}{12}M$$

$$0.64 m_{\text{man}}/2 = \tfrac{1}{12}M(1 - 0.64)$$

$$M = \left(\frac{12}{0.36}\right)\left(\frac{0.64}{2}\right) m_{\text{man}} = \left(\frac{96}{9}\right) m_{\text{man}}$$

$$M = \left(\frac{96}{9}\right)(75\ kg) = 800\ kg$$