Chapter 12 Problem 14 †

Given
$\vec{F}_1 = \{2\hat{\imath} + 2\hat{\jmath}\}\ N$ applied at $\vec{r}_1 = \{2\hat{\imath}\}\ m$
$\vec{F}_2 = \{-2\hat{\imath} - 3\hat{\jmath}\}\ N$ applied at $\vec{r}_2 = \{-1\hat{\imath}\}\ m$
$\vec{F}_3 = \{1\hat{\jmath}\}\ N$ applied at $\vec{r}_3 = \{-7\hat{\imath} + 1\hat{\jmath}\}\ m$

Solution
a) Show that the net force is zero.

Adding the three forces together gives

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \{2\hat{\imath} + 2\hat{\jmath}\}\ N + \{-2\hat{\imath} - 3\hat{\jmath}\}\ N + \{1\hat{\jmath}\}\ N$$

$$\vec{F}_{tot} = \{(2-2)\hat{\imath} + (2-3+1)\hat{\jmath}\}\ N = 0$$

b) Show the net torque about the origin is zero.

$$\vec{\tau}_{tot} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3$$

$$\vec{\tau}_{tot} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 0 \\ 2 & 2 & 0 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 0 & 0 \\ -2 & -3 & 0 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -7 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\vec{\tau}_{tot} = 4\hat{k} + 3\hat{k} - 7\hat{k} = 0$$

c) Show the net torque about $(3\ m, 2\ m)$ and $(-7\ m, 1\ m)$ is zero.

The new position vectors for $(3\ m, 2\ m)$ will be

$$\vec{r}_1' = \vec{r}_1 - \vec{r}_p = \{2\hat{\imath}\}\ m - \{3\hat{\imath} + 2\hat{\jmath}\}\ m = \{-1\hat{\imath} - 2\hat{\jmath}\}\ m$$

$$\vec{r}_2' = \vec{r}_2 - \vec{r}_p = \{-1\hat{\imath}\}\ m - \{3\hat{\imath} + 2\hat{\jmath}\}\ m = \{-4\hat{\imath} - 2\hat{\jmath}\}\ m$$

$$\vec{r}_3' = \vec{r}_3 - \vec{r}_p = \{-7\hat{\imath} + 1\hat{\jmath}\}\ m - \{3\hat{\imath} + 2\hat{\jmath}\}\ m = \{-10\hat{\imath} - 1\hat{\jmath}\}\ m$$

The torque is then

$$\vec{\tau}_{tot}' = \vec{\tau}_1' + \vec{\tau}_2' + \vec{\tau}_3' = \vec{r}_1' \times \vec{F}_1 + \vec{r}_2' \times \vec{F}_2 + \vec{r}_3' \times \vec{F}_3$$

$$\vec{\tau}_{tot}' = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & -2 & 0 \\ 2 & 2 & 0 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & -2 & 0 \\ -2 & -3 & 0 \end{vmatrix} + \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -10 & -1 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\vec{\tau}_{tot}' = 2\hat{k} + 8\hat{k} - 10\hat{k} = 0$$

The new position vectors for $(-7\ m, 1\ m)$ will be

$$\vec{r}_1' = \vec{r}_1 - \vec{r}_p = \{2\hat{\imath}\}\ m - \{-7\hat{\imath} + 1\hat{\jmath}\}\ m = \{9\hat{\imath} - 1\hat{\jmath}\}\ m$$

$$\vec{r}_2' = \vec{r}_2 - \vec{r}_p = \{-1\hat{\imath}\}\ m - \{-7\hat{\imath} + 1\hat{\jmath}\}\ m = \{6\hat{\imath} - 1\hat{\jmath}\}\ m$$

$$\vec{r}_3' = \vec{r}_3 - \vec{r}_p = \{-7\hat{\imath} + 1\hat{\jmath}\}\ m - \{-7\hat{\imath} + 1\hat{\jmath}\}\ m = \{0\hat{\imath} + 0\hat{\jmath}\}\ m$$

† Problem from Essential University Physics, Wolfson

The torque is then

$$\vec{\tau}'_{tot} = \vec{\tau}'_1 + \vec{\tau}'_2 + \vec{\tau}'_3 = \vec{r}'_1 \times \vec{F}_1 + \vec{r}'_2 \times \vec{F}_2 + \vec{r}'_3 \times \vec{F}_3$$

$$\vec{\tau}'_{tot} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 9 & -1 & 0 \\ 2 & 2 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -1 & 0 \\ -2 & -3 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\vec{\tau}'_{tot} = 20\hat{k} - 20\hat{k} + 0\hat{k} = 0$$

About any point (x, y) in general the torque is zero.

$$\vec{r}'_1 = \vec{r}_1 - \vec{r}_p = \{2\hat{i}\}\ m - \{x\hat{i} + y\hat{j}\}\ m = \{(2-x)\hat{i} - y\hat{j}\}\ m$$

$$\vec{r}'_2 = \vec{r}_2 - \vec{r}_p = \{-1\hat{i}\}\ m - \{x\hat{i} + y\hat{j}\}\ m = \{(-1-x)\hat{i} - y\hat{j}\}\ m$$

$$\vec{r}'_3 = \vec{r}_3 - \vec{r}_p = \{-7\hat{i} + 1\hat{j}\}\ m - \{x\hat{i} + y\hat{j}\}\ m = \{(-7-x)\hat{i} + (1-y)\hat{j}\}\ m$$

The torque is then

$$\vec{\tau}'_{tot} = \vec{\tau}'_1 + \vec{\tau}'_2 + \vec{\tau}'_3 = \vec{r}'_1 \times \vec{F}_1 + \vec{r}'_2 \times \vec{F}_2 + \vec{r}'_3 \times \vec{F}_3$$

$$\vec{\tau}'_{tot} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2-x & -y & 0 \\ 2 & 2 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1-x & -y & 0 \\ -2 & -3 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -7-x & 1-y & 0 \\ 0 & 1 & 0 \end{vmatrix}$$

$$\vec{\tau}'_{tot} = (2(2-x) + 2y)\hat{k} + (-3(-1-x) - 2y)\hat{k} + (-7-x)\hat{k} = 0$$

$$\vec{\tau}_{tot} = (4 - 2x + 2y + 3 + 3x - 2y - 7 - x)\hat{k} = 0$$