Chapter 11 Problem 18 [†]

Given
$\vec{F} = \{1.3\hat{\imath} + 2.7\hat{\jmath}\}\ N$
$\vec{r} = \{3.0\hat{\imath} + 0\hat{\jmath}\}\ m$

Solution
Find the torque about the origin.

Torque is given by the cross product between the force arm and the force vector.

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3.0 & 0 & 0 \\ 1.3 & 2.7 & 0 \end{vmatrix}\ N \cdot m$$

Expanding the matrix by minors gives

$$\vec{\tau} = \left(\hat{\imath} \begin{vmatrix} 0 & 0 \\ 2.7 & 0 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 3.0 & 0 \\ 1.3 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 3.0 & 0 \\ 1.3 & 2.7 \end{vmatrix} \right)\ N \cdot m$$

Solving the determinant of the 2×2 matrices gives

$$\vec{\tau} = \{\hat{\imath}((0)(0) - (2.7)(0)) - \hat{\jmath}((3)(0) - (1.3)(0)) + \hat{k}((3.0)(2.7) - (1.3)(0))\}\ N \cdot m$$

$$\vec{\tau} = \{\hat{\imath}(0) - \hat{\jmath}(0) + \hat{k}(8.1)\}\ N \cdot m$$

$$\vec{\tau} = 8.1\hat{k}\ N \cdot m$$

b) Find the torque about the point $\vec{r}_0 = \{-1.3\hat{\imath} + 2.4\hat{\jmath}\}\ m$.

The force arm is the difference between the point of interest and the location at which the force is applied. Therefore,

$$\vec{\tau} = (\vec{r} - \vec{r}_0) \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x - r_{0x} & r_y - r_{0y} & r_z - r_{0z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3.0 - (-1.3) & 0 - 2.4 & 0 \\ 1.3 & 2.7 & 0 \end{vmatrix}\ N \cdot m$$

$$\vec{\tau} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4.3 & -2.4 & 0 \\ 1.3 & 2.7 & 0 \end{vmatrix}\ N \cdot m$$

Expanding the matrix by minors gives

$$\vec{\tau} = \left(\hat{\imath} \begin{vmatrix} -2.4 & 0 \\ 2.7 & 0 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 4.3 & 0 \\ 1.3 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 4.3 & -2.4 \\ 1.3 & 2.7 \end{vmatrix} \right)\ N \cdot m$$

[†]Problem from Essential University Physics, Wolfson

Solving the determinant of the 2×2 matrices gives

$$\vec{\tau} = \{\hat{i}((-2.4)(0) - (2.7)(0)) - \hat{j}((4.3)(0) - (1.3)(0)) + \hat{k}((4.3)(2.7) - (1.3)(-2.4))\} \; N \cdot m$$

$$\vec{\tau} = \{\hat{i}(0) - \hat{j}(0) + \hat{k}(14.7)\} \; N \cdot m$$

$$\vec{\tau} = 14.7\hat{k} \; N \cdot m$$