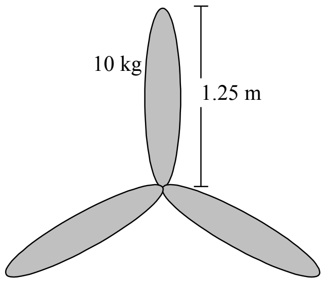
Chapter 10   Problem 52 †



Given
$m = 10\ kg$
$l = 1.25\ m$

Solution
a) Find the moment of inertia of the propeller.

Treating each blade as a rod, the total moment of inertia is three times the moment of inertia for a rod rotated about one end.

$$I_{tot} = 3I_{rod} = 3(\tfrac{1}{3}MR^2) = MR^2 = (10\ kg)(1.25\ m)^2$$

$$I_{tot} = 15.6\ kg \cdot m^2$$

b) Find the time to increase the angular speed from 1400 rmp to 1900 rmp due to a torque of 2700 $N \cdot m$.

First convert the angular speeds into rad/s.

$$\omega_0 = \left(\frac{1400\ rev}{min}\right)\left(\frac{2\pi\ rad}{rev}\right)\left(\frac{1\ min}{60\ s}\right) = 147\ rad/s$$

$$\omega_f = \left(\frac{1900\ rev}{min}\right)\left(\frac{2\pi\ rad}{rev}\right)\left(\frac{1\ min}{60\ s}\right) = 199\ rad/s$$

Angular acceleration is related to torque by the formula

$$\tau = I\alpha$$

Solving for angular acceleration gives

$$\alpha = \frac{\tau}{I} \tag{1}$$

Use the definition of average angular acceleration to get the time.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} \tag{2}$$

†Problem from Essential University Physics, Wolfson

Combining equation 1 and 2 gives a time of

$$\Delta t = \frac{\Delta\omega}{\tau/I} = \frac{I(\omega_f - \omega_0)}{\tau}$$

Substituting in the values gives

$$\Delta t = \frac{(15.6\ kg \cdot m^2)(199\ rad/s - 147\ rad/s)}{2700\ N \cdot m} = 0.30\ s$$