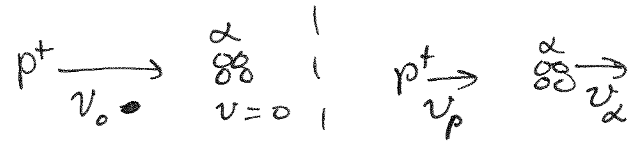
Chapter 9 Problem 58

$v_o = 3.0 \times 10^6$ m/s



m_p — mass of proton

$m_\alpha = 4 m_p$ — mass of alpha particle

(Elastic Collision)

What % of kinetic energy does the proton retain?

First we need to find the particle's velocity after the collision

By Conservation of momentum

$$P_o = P_f$$

$$m_p \cdot v_o + 0 = m_p v_p + m_\alpha v_\alpha$$

Given $m_\alpha = 4 m_p$,

$$m_p v_o = m_p v_p + 4 m_p v_\alpha \rightarrow \boxed{v_o = v_p + 4 v_\alpha} \quad \#1$$

By Conservation of Energy (elastic collision)

$$K_o = K_f$$

$$\frac{1}{2} m_p v_o^2 + 0 = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_\alpha v_\alpha^2$$

Given $m_\alpha = 4 m_p \rightarrow \frac{1}{2} m_p v_o^2 = \frac{1}{2} m_p v_p^2 + \frac{1}{2}(4 m_p) v_\alpha^2$

$$\boxed{v_o^2 = v_p^2 + 4 v_\alpha^2} \quad \#2$$

Using #1 solve for $v_\alpha \longrightarrow v_\alpha = \frac{v_o - v_p}{4}$

Substitute into #2

$$v_o^2 = v_p^2 + 4\left[\frac{v_o - v_p}{4}\right]^2$$

$$v_o^2 = v_p^2 + 4\left[\frac{v_o^2 - 2 v_o v_p + v_p^2}{16}\right]$$

$$v_o^2 = v_p^2 + \frac{v_o^2 - 2 v_o v_p + v_p^2}{4}$$

$$0 = v_p^2 + \frac{v_0^2}{4} - \frac{v_0 v_p}{2} + \frac{v_p^2}{4} - v_0^2$$

$$0 = v_p^2\left(1+\frac{1}{4}\right) - \frac{v_0 v_p}{2} - v_0^2\left(1-\frac{1}{4}\right)$$

$$0 = 1.25\, v_p^2 - 0.5\, v_0 v_p - 0.75\, v_0^2$$

Since v_0 is known, solve for v_p using the Quadratic formula

$$v_p = \frac{0.5 v_0 \pm \sqrt{(-0.5 v_0)^2 - 4(1.25)(-0.75 v_0^2)}}{2(1.25)}$$

$$v_p = \frac{0.5 v_0 \pm v_0\sqrt{(0.5)^2 + 4(1.25)(0.75)}}{2.50}$$

$$= \frac{0.5 v_0 \pm v_0 \cdot 2}{2.50}$$

2 solutions are

$$v_p = \frac{0.5 v_0 + 2 v_0}{2.50} = \frac{2.5 v_0}{2.5} = v_0$$ (No interaction with the alpha particle.)

$$v_p = \frac{0.5 v_0 - 2 v_0}{2.50} = \frac{-1.5 v_0}{2.50} = \boxed{-0.60\, v_0}$$ (This is the solution we want)

[Although not needed

$$v_p = -0.60(3\times10^6 \text{ m/s}) = -1.8\times10^6 \text{ m/s}$$

$$v_\alpha = \frac{v_0 - v_p}{4} = \frac{v_0 - (-0.60)v_0}{4} = \frac{1.6}{4} v_0 = 0.40\, v_0 = 1.2\times10^6 \text{ m/s}$$]

% ~~Loss of~~ Retained Kinetic energy $K_0 = \frac{1}{2} m_p v_0^2$ $K_f = \frac{1}{2} m_p v_p^2$

$$\%\text{ Retained} = \frac{K_f}{K_0}\times100\% = \frac{\frac{1}{2} m_p v_p^2 \times 100\%}{\frac{1}{2} m_p v_0^2} = \frac{v_p^2 \times 100\%}{v_0^2} = \frac{(0.60 v_0)^2 \times 100\%}{v_0^2}$$

$$= 0.36 \times 100\%$$

$$= \boxed{36\%}$$