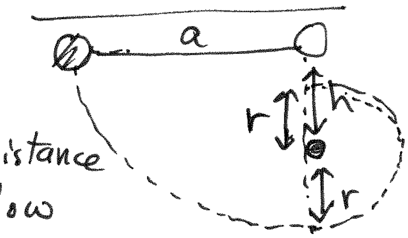
Chapter 8 Problem 75

A ball is dropped and the cord swings it to the right. The distance ~~height~~ of the peg below the pivot point is h.



If the ball swings around the peg, $h = \frac{3a}{5}$. Show this is true.

The ball travels around the peg with radius, r.

Since the peg is a distance h below the pivot, then

$$\boxed{a = h + r} \quad (\#1)$$

When the ball is at the top of its circle, the centripetal acceleration needs to be as great as the acceleration due to gravity. Otherwise, the string attached to the ball will go slack.

$$\therefore a_c = g \rightarrow \frac{v^2}{r} = g \rightarrow \boxed{v^2 = g \cdot r} \quad (\#2)$$

If the ball starts horizontal to the left of the pivot it is a distance, 'a' above the lowest part of the motion of the ball. Potential energy

$$\boxed{U_0 = mga} \quad (\#3)$$

When the ball is at the top of the circle, it is a distance of 2r above the lowest part of the ball's motion

$$\therefore \text{Potential energy } \boxed{U_1 = mg2r} \quad (\#4)$$

The ball initially starts at rest, so $K_0 = 0$

The kinetic energy at the top of the circle is

$$\boxed{K_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2} mgr} \quad (\#5)$$

see eq (#2)

Now use conservation of Energy

$$E_0 = K_0 + U_0 = K_1 + U_1$$

$$0 + mga = \tfrac{1}{2} mgr + 2mgr \Rightarrow mga = 2.5\,mgr$$

eq (#3), eq (#5), eq (#4)

from (#1) $r = a - h$ so

$$mga = 2.5\,mg(a - h)$$

$$mga = 2.5\,mga - 2.5\,mgh$$

$$2.5\,mgh = 1.5\,mga \longrightarrow h = \frac{1.5\,mga}{2.5\,mg}$$

$$h = \frac{1.5}{2.5} a$$

$$* \boxed{h = \frac{3}{5} a}$$