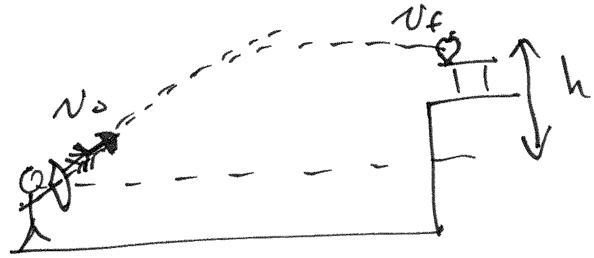
Chapter 8 — Problem 55

$m = 0.0200\,kg$

$k = 330\,\frac{N}{m}$

$\Delta x = 0.55\,m$

$h = 5.00\,m$



a) At what speed does the arrow leave the bow?
Assume all of the energy of the bow goes into the kinetic energy of the arrow.

$$E_0 = K_0 + U_0 = 0 + \tfrac{1}{2}kx^2 \Leftrightarrow E_f = K_f + U_f = \tfrac{1}{2}mv^2 + 0$$

- 0: Arrow initially at rest
- $\tfrac{1}{2}kx^2$: Potential energy in the bow
- $\tfrac{1}{2}mv^2$: all kinetic energy in the arrow
- 0: no potential in the bow.

Then $E_0 = E_f$

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv^2 \rightarrow kx^2 = mv^2 \rightarrow v^2 = \frac{k}{m}x^2$$

$$v_0 = \sqrt{\frac{k}{m}}\,x = \sqrt{\frac{330\,\frac{N}{m}}{0.0200\,kg}}\,(0.55\,m)$$

$$\boxed{v_0 = 70.6\,m/s}$$

b) What is the speed at the apple?
Assume no energy loss due to air drag.
Then the potential energy gain of the arrow as it reaches the apple is $U_f = mgh$
The initial potential energy is zero.

$$\therefore \quad E_0 = E_f \longrightarrow K_0 + U_0 = K_f + U_f$$

$$\tfrac{1}{2}mv_0^2 + 0 = \tfrac{1}{2}mv_f^2 + mgh$$

This is now the energy as the arrow leaves the bow

With Algebra

$$\tfrac{1}{2}mv_0^2 - mgh = \tfrac{1}{2}mv_f^2$$

$$v_f = \sqrt{v_0^2 - 2gh} = \sqrt{(70.6)^2 - 2(9.8)(5.0)}$$

$$\boxed{v_f = 69.9\,m/s}$$