

$$v_{\min} = 15.0 \text{ m}$$

- a) What is the speed of the roller coaster at the top if the downward acceleration is $1.50g$

The free-body diagram of the coaster at the top of the curve is

The track exerts a force and gravity is exerting a force

By Newton's 2nd law

$$\sum \vec{F} = m\vec{a}$$

$$\vec{N} + \vec{W} = m\vec{a}$$

$$-N\hat{j} - mg\hat{j} = -ma\hat{j} \quad \therefore N + mg = ma$$

$$a = g + \frac{N}{m}$$

~~The normal force is providing centripetal~~

Since the curvature of the loop is providing centripetal acceleration, the forces providing this centripetal acceleration is gravity and the track's normal force.

Regardless, the problem states that the acceleration is $1.5g$ in the downward direction.

Centripetal acceleration is given by

$$a_c = \frac{v^2}{r} \rightarrow v = \sqrt{a_c \cdot r}$$

$$= \sqrt{1.5g \cdot r}$$

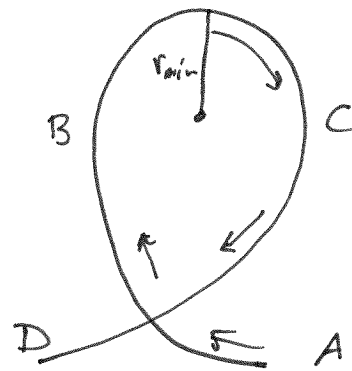
$$= \sqrt{1.5 (9.8 \text{ m/s}^2) (15.0 \text{ m})}$$

$$= \boxed{14.8 \text{ m/s}}$$

Note: $a = g + \frac{N}{m} = 1.5g$

$$\therefore \frac{N}{m} = \frac{1}{2}g$$

The person feels like they are pressed into their seat by half of their weight.



b) How high above the top of the loop must the roller coaster start from rest assuming negligible friction?

Although this could be answered using conservation of energy (chapter 8), we can use the kinematic equations to answer this question.

First, you need to realize that normal forces act perpendicular to the direction of motion. Therefore, they do not speed up or slow down the roller coaster (Normal forces do no work!), but they redirect the motion of the coaster.

As a result, only gravity results in increasing the speed of the roller coaster.

If $v_0 = 0 \text{ m/s}$ and $v_f = 14.8 \text{ m/s}$ (speed at the top of the loop)

Then the 4th kinematic equation can be used.

$$v_f^2 - v_0^2 = 2a\Delta y$$

$$\Delta y = \frac{v_f^2 - v_0^2}{2a}$$

$$= \frac{(14.8 \text{ m/s})^2 - 0^2}{2(9.8 \text{ m/s}^2)}$$

(we will ignore the negative sign because the normal force will redirect it from going downward to going around the top of the loop.)

$$\Delta y = -11.2 \text{ m}$$

Needs to drop 11.2 m to the height of the loop's top.

Again this is more obvious with conservation of energy.)

c) If it starts 5.0 m higher than this, how much energy is lost to friction?

[Chapter 8: Gravitational potential energy is $\Delta U = mg\Delta h = (1.50 \times 10^3 \text{ kg})(9.8 \text{ m/s}^2)(5 \text{ m}) = 73500 \text{ J}$]