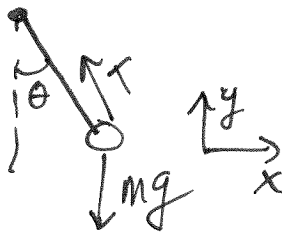
Chapter 6

Problem 120

At what angle relative to vertical does the plumb bob hang?

$$v = 90.0\ \frac{km}{h}\left(\frac{1\ hr}{3600\ s}\right)\left(\frac{1000\ m}{1\ km}\right) = 25\ m/s$$



$r = 300.0\ m$

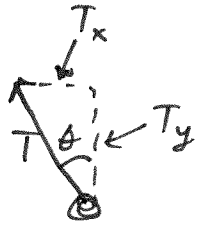
The centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(25\ m/s)^2}{300\ m} = 2.08\ m/s^2$$

From the Free-Body diagram

$$\sum \vec{F} = m\vec{a}$$

$$\vec{T} + \vec{W} = m\vec{a}$$



using the x-axis as to the right horizontally

$$\vec{T} = -T\sin\theta\,\hat{i} + T\cos\theta\,\hat{j}$$

$$\vec{W} = -mg\,\hat{j}$$

$$\vec{a} = -a\,\hat{i} \leftarrow [\text{centripetal acceleration is toward the center of the track.}]$$

Then

$$-T\sin\theta\,\hat{i} + T\cos\theta\,\hat{j} - mg\,\hat{j} = -ma\,\hat{i}$$

x-dir $\quad -T\sin\theta = -ma \qquad (\#1)$

y-dir $\quad T\cos\theta - mg = 0 \qquad (\#2)$

Take (#2) and solve for Tension

$$T\cos\theta = mg \longrightarrow T = \frac{mg}{\cos\theta}$$

Sub into (#1)

$$-\frac{mg}{\cos\theta}\sin\theta = -ma \longrightarrow mg\tan\theta = ma$$

$$\tan\theta = \frac{ma}{mg} = \frac{a}{g} \quad \therefore \theta = \tan^{-1}\left(\frac{a}{g}\right) = \tan^{-1}\left(\frac{2.08\ m/s^2}{9.80\ m/s^2}\right)$$

$$\boxed{\theta = 12.0^\circ}$$