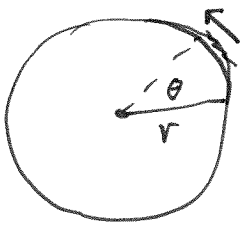
Chapter 6 — Problem 116

$m = 0.50\,kg$

$\vec{r} = 4.0\cos(3t)\,\hat{i} + 4.0\sin(3t)\,\hat{j}$




$\theta = 3t$

$r = 4.0\,m$

With this equation we see the radius is 4.0 m and the angle changes at a rate of $3t$. Since time is in seconds, the value of '3' is an angular velocity with units of radians/second

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a) Find the velocity and acceleration as functions of time.

velocity is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left[4.0\cos(3t)\,\hat{i} + 4.0\sin(3t)\,\hat{j}\right]$$

$$\boxed{\vec{v} = -12.0\sin(3t)\,\hat{i} + 12.0\cos(3t)\,\hat{j}}$$

Note: This is tangent to the curve, with a velocity of $v = 12.0\,\frac{m}{s}$

acceleration is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[-12.0\sin(3t)\,\hat{i} + 12.0\cos(3t)\,\hat{j}\right]$$

$$\boxed{\vec{a} = -36.0\cos(3t)\,\hat{i} - 36.0\sin(3t)\,\hat{j}}$$

b) Show acceleration vector is towards the center of the circle.

The $\vec{r}$ is always pointing from the center of the circle to the surface of the circle.

Compare the functions $\vec{r}$ and $\vec{a}$. They have the same functionality cosine in $\hat{i}$ and sine in $\hat{j}$. The differences are in the magnitude $36.0\,m/s^2$ rather than 4.0 m and the direction (there is a negative sign.)

The negative sign means it points to the center of the circle.

c) Find the centripetal force vector [The force that causes the centripetal acceleration!]

$$\vec{F} = m\vec{a} = (0.50\,kg)(-36.0\cos(3t)\,\hat{i} - 36.0\sin(3t)\,\hat{j})$$

$$\boxed{\vec{F} = -18.0\cos(3t)\,\hat{i} - 18.0\sin(3t)\,\hat{j}}$$

Force is in newtons.