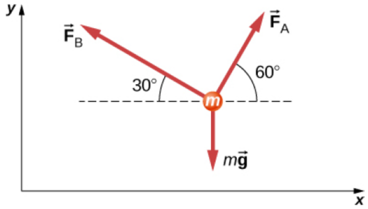
Chapter 5 Problem 82 [†]



Given
$m = 1.0\ kg$
$\vec{a} = -20\hat{\imath}\ m/s^2$

Solution
What are $\vec{F}_A$ and $\vec{F}_B$.

By Newton's 2nd law

$$\vec{F}_{net} = m\vec{a}$$

Substituting in the three forces acting on the system gives

$$\vec{F}_A + \vec{F}_B + \vec{W} = m\vec{a} \qquad Eq(1)$$

Resolve each of the forces and acceleration into the coordinate system provided in the diagram.

$$\vec{F}_A = F_A\cos(60°)\hat{\imath} + F_A\sin(60°)\hat{\jmath}$$

$$\vec{F}_B = -F_B\cos(30°)\hat{\imath} + F_B\sin(30°)\hat{\jmath}$$

$$\vec{W} = -mg\hat{\jmath}$$

$$\vec{a} = -a\hat{\imath}$$

where $a = 20\ m/s^2$. Inserting these values into equation (1) gives

$$F_A\cos(60°)\hat{\imath} + F_A\sin(60°)\hat{\jmath} - F_B\cos(30°)\hat{\imath} + F_B\sin(30°)\hat{\jmath} - mg\hat{\jmath} = -ma\hat{\imath}$$

The x-component of this equation is

$$F_A\cos(60°) - F_B\cos(30°) = -ma \qquad Eq(2)$$

The y-component of this equation is

$$F_A\sin(60°) + F_B\sin(30°) - mg = 0 \qquad Eq(3)$$

We now have two equations with two unknowns. Take equation (2) and solve for F_A.

$$F_A\cos(60°) = -ma + F_B\cos(30°)$$

$$F_A = \frac{-ma + F_B\cos(30°)}{\cos(60°)} \qquad Eq(4)$$

Substitute this result into equation (3)

$$\left(\frac{-ma + F_B\cos(30°)}{\cos(60°)}\right)\sin(60°) + F_B\sin(30°) - mg = 0$$

[†]Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

Now solve for F_B.

$$(-ma + F_B\cos(30°))\tan(60°) + F_B\sin(30°) - mg = 0$$

$$-ma\tan(60°) + F_B\cos(30°)\tan(60°) + F_B\sin(30°) - mg = 0$$

$$F_B\cos(30°)\tan(60°) + F_B\sin(30°) = mg + ma\tan(60°)$$

$$F_B\left(\cos(30°)\tan(60°) + \sin(30°)\right) = m(g + a\tan(60°))$$

$$F_B = \frac{m(g + a\tan(60°))}{\cos(30°)\tan(60°) + \sin(30°)}$$

Substituting in the appropriate values gives

$$F_B = \frac{(1.0\ kg)(9.80\ m/s^2 + (20\ m/s^2)\tan(60°))}{\cos(30°)\tan(60°) + \sin(30°)} = \frac{44.4}{2.0}\ N$$

$$F_B = 22.2\ N$$

Use this value and substitute into Eq (4) to solve for F_A

$$F_A = \frac{-(1.0\ kg)(20.0\ m/s^2) + (22.2\ N)\cos(30°)}{\cos(60°)}$$

$$F_A = \frac{-0.774\ N}{0.50} = -1.55\ N$$

This means F_A is pointing in the opposite direction from the diagram (into the 3rd quadrant.)