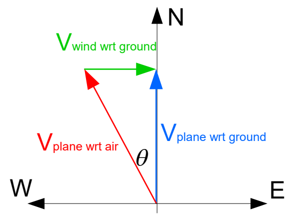
Chapter 4 Problem 76 [†]



Given

$v_{plane\ wrt\ air} = 200\ km/hr$
$\vec{v}_{wind\ wrt\ ground} = 50\ \hat{i}\ km/hr$

Solution

a) In what direction should the pilot direct the plane in order to go due north?

In Galilean transform problems, you need to identify three things. 1) The object of interest (P), 2) The stationary frame (S) and 3) the moving frame (S'). In this case, the plane is the object of interest, the ground is the stationary frame and the air is the moving frame. From the equation

$$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$$

The wind is a measure of the air's motion relative to the ground. Therefore,

$$\vec{v}_{S'S} = \vec{v}_{wind\ wrt\ ground} = 50\ \hat{i}\ km/hr$$

The desired motion of the plane relative to the ground is due north. Since we don't know the magnitude of this velocity, we will use V_N.

$$\vec{v}_{PS} = \vec{v}_{plane\ wrt\ ground} = V_N\ \hat{j}$$

As illustated in the diagram, the plane's motion relative to the air must be directed west of north. That way the wind will redirect the plane so it travels due north. The velocity of the plane relative to the air will have an x-component in the negative direction whose magnitude corresponds to the opposite side to the given angle. The y-component is positive and corresponds to the adjacent side to the given angle. Therefore, this vector is

$$\vec{v}_{PS'} = \vec{v}_{plane\ wrt\ air} = -(200\ km/hr)\sin\theta\hat{i} + (200\ km/hr)\cos\theta\hat{j}$$

Substitute these values into the Galilean transform gives

$$\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$$

$$(V_N\ \hat{j}) = (-(200\ km/hr)\sin\theta\hat{i} + (200\ km/hr)\cos\theta\hat{j}) + (50\ \hat{i}\ km/hr)$$

From this equation, the equation in the x-direction is

$$0 = -200\sin\theta + 50 \qquad Eq.(1)$$

Notice, I dropped the units for clarity. The equation in the y-direction is

$$V_N = 200\cos\theta \qquad Eq.(2)$$

[†] Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

Use equation (1) to find the angle the plane must head west of north.

$$200 \sin \theta = 50$$

$$\theta = \sin^{-1}\left(\frac{50}{200}\right)$$

$$\theta = 14.5^\circ$$

b) How long does it take her to reach a point 300 km directly north of her starting point.

Since we have the angle, we can use equation (2) to calculate the velocity of the plane relative to the ground.

$$V_N = 200 \cos \theta$$

$$V_N = 200 \cos 14.5^\circ = 194\ km/hr$$

Using the definition of velocity

$$v = \frac{y}{t}$$

Solving for time gives

$$t = \frac{y}{v} = \frac{300\ km}{194\ km/hr} = 1.55\ hr$$