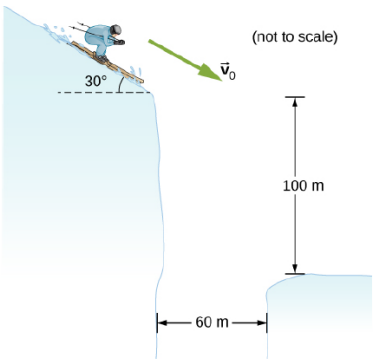
Chapter 4 Problem 46 †



Given

$v_0 = 60\ km/h$
$\theta = -30°$
$y_f = -100\ m$
$x_f = 60\ m$
$a_y = -g = -9.80\ m/s^2$

Solution

Does the agent clear the gorge?

First convert the speed into m/s and break into scalar components using unit vectors.

$$v_0 = \frac{60\ km}{1\ hr}\left(\frac{1000\ m}{1\ km}\right)\left(\frac{(1\ hr)}{3600\ s}\right) = 16.7\ m/s$$

$$\vec{v}_0 = v_0 \cos\theta\ \hat{i} + v_0 \sin\theta\ \hat{j}$$
$$\vec{v}_0 = (16.7\ m/s)\cos(-30°)\ \hat{i} + (16.7\ m/s)\sin(-30°)\ \hat{j} = \{14.5\ \hat{i} - 8.35\ \hat{j}\}\ m/s$$

From the initial values, the position vector is

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

Set the coordinate system at the point where the agent begins the jump over the gorge. Therefore, $\vec{r}_0 = 0$.

$$\vec{r} = 0 + \left\{v_{x0}\ \hat{i} + v_{y0}\ \hat{j}\right\} t + \tfrac{1}{2}\left\{-g\ \hat{j}\right\} t^2$$

Regrouping gives

$$\vec{r} = \left\{[(v_{x0}t]\ \hat{i} + [v_{y0}t - \tfrac{1}{2}gt^2]\ \hat{j}\right\}$$

Since $\vec{r} = x_f\ \hat{i} + y_f\ \hat{j}$, we get an equation for the x-direction and an equation for the y-direction.

$$x_f = v_{x0}t$$

$$y_f = v_{y0}t - \tfrac{1}{2}gt^2$$

Use the first equation to find the time needed to clear the width of the gorge.

$$t = \frac{x_f}{v_{x0}} = \frac{60\ m}{14.5\ m/s} = 4.14\ s$$

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†Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

During this time the vertical position is

$$y_f = (-8.35\ m/s)(4.14\ s) - \tfrac{1}{2}(9.80\ m/s^2)(4.14\ s)^2 = -34.6\ m - 84.0\ m = -118.6\ m$$

Since the agent drops 119 m by the time he moved 60 m horizontally, he is below the snow on the other side, which is 100 m below the ledge from which he left.