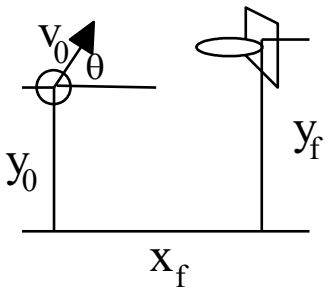
Chapter 4 Problem 40 [†]



Given
$y_0 = 1.8\ m$
$y_f = 3.0\ m$
$x_0 = 0\ m$
$x_f = 6.1\ m$
$\theta = 60°$
$a_y = -g = -9.80\ m/s^2$

Solution
Find the initial speed if the ball is to go through the basket.

From the initial values, the position vector is

$$\vec{r} = \vec{r_0} + \vec{v_0}t + \tfrac{1}{2}\vec{a}t^2$$

$$\vec{r} = y_0\ \hat{j} + \left\{v_0\ \cos\theta\ \hat{i} + v_0\ \sin\theta\ \hat{j}\right\} t + \tfrac{1}{2}\left\{-g\ \hat{j}\right\} t^2$$

Regrouping gives

$$\vec{r} = \left\{[(v_0\ \cos\theta t]\ \hat{i} + [y_0 + v_0\ \sin\theta t - \tfrac{1}{2}gt^2]\ \hat{j}\right\}$$

When the ball reaches the hoop, the x-component gives the equation

$$x_f = v_0 \cos\theta t$$

Solving for t gives

$$t = \frac{x_f}{v_0 \cos\theta}$$

When the ball reaches the hoop, the y-component gives the equation

$$y_f = y_0 + v_0 \sin\theta t - \tfrac{1}{2}gt^2$$

Substitute in the equation obtained from the x-component to eliminate time dependence.

$$y_f = y_0 + v_0 \sin\theta \left(\frac{x_f}{v_0 \cos\theta}\right) - \tfrac{1}{2}g \left(\frac{x_f}{v_0 \cos\theta}\right)^2$$

[†]Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

Simplifying gives

$$y_f = y_0 + x_f \tan\theta - \frac{g x_f^2}{2 v_0^2 \cos^2\theta}$$

Now solve for v_0

$$y_f - y_0 - x_f \tan\theta = -\frac{g x_f^2}{2 v_0^2 \cos^2\theta}$$

$$x_f \tan\theta + y_0 - y_f = \frac{g x_f^2}{2 v_0^2 \cos^2\theta}$$

$$(x_f \tan\theta + y_0 - y_f) v_0^2 = \frac{g x_f^2}{2\cos^2\theta}$$

$$v_0^2 = \frac{g x_f^2}{2(x_f \tan\theta + y_0 - y_f)\cos^2\theta}$$

$$v_0 = \sqrt{\frac{g x_f^2}{2(x_f \tan\theta + y_0 - y_f)\cos^2\theta}}$$

Now substitute in the values for the known variables

$$v_0 = \sqrt{\frac{(9.8\ m/s^2)(6.1\ m)^2}{2((6.1\ m)\tan 60^\circ + 1.8\ m - 3.0\ m)\cos^2 60^\circ}}$$

$$v_0 = \sqrt{\frac{365\ m^3/s^2}{2(10.6\ m + 1.8\ m - 3.0\ m)\cos^2 60^\circ}}$$

$$v_0 = \sqrt{\frac{365\ m^3/s^2}{2(9.4\ m)\cos^2 60^\circ}}$$

$$v_0 = \sqrt{\frac{365\ m^3/s^2}{4.7\ m}}$$

$$v_0 = 8.8\ m/s$$