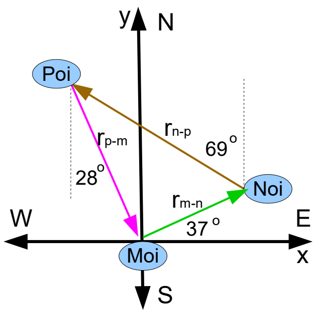
Chapter 2 Problem 75 †



Given

$\vec{r}_{m-n} = 4.76\ nmi\ \angle 37°$

$\vec{r}_{n-p} = A \angle 69°$ west of north

$\vec{r}_{p-m} = B \angle 28°$ east of south

$1.000\ nmi = 1852\ m = 1.852\ km$

Solution

What are the distances between the three islands?

The distance from Moi to Noi is given at 4.76 nmi. Converting this to kilometers gives

$$r_{m-n} = 4.76\ nmi \left(\frac{1.852\ km}{1.000\ nmi}\right) = 8.82\ km$$

The other two distances are unknown, but we know the boat gets back home and the headings it took on its travels. Begin by breaking each vector into component scalars.

Begin with r_{m-n}.

$$\vec{r}_{m-n} = \{4.76\cos(37°)\hat{i} + 4.76\sin(37°)\hat{j}\}\ nmi = \{3.802\hat{i} + 2.865\hat{j}\}\ nmi$$

Going from Noi to Poi, the boat travels 69° west of north. I will use the given angle to calculate the components. A right triangle can be formed by using the vertical line from Noi and drawing a horizontal line from Poi. The hypotenus of this triangle is the path taken by the boat. The vertical line corresponds to the y-axis and is the adjacent side to the 69° angle. Instead of using sine for the y-component, we will use cosine. Likewise the opposite side corresponds to the x-axis and, therefore, sine should be used.

$$\vec{r}_{n-p} = -A\sin(69°) + A\cos(69°) = -0.9336\ A\ \hat{i} + 0.3584\ A\ \hat{j}$$

Since the boat is traveling in the direction of the 2nd quadrant, the x-component is negative and the y-component is positive.

Going from Poi to Moi, the boat travels 28° east of south. Once again we can form a triangle where the opposite side corresponds to the x-axis and the adjacent side to the y-axis. This time the boat is traveling towards the 4th quadrant. The components are then

$$\vec{r}_{p-m} = B\sin(28°) - B\cos(28°) = 0.4695\ B\ \hat{i} - 0.8829\ B\ \hat{j}$$

We are now ready to solve for the unknown distances.

†Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

Since the boat ends up where it started, the sum of all three vectors equals zero.

$$\vec{r}_{m-n} + \vec{r}_{n-p} + \vec{r}_{p-m} = 0$$

$$\{3.802\hat{\imath} + 2.865\hat{\jmath}\}\ nmi + \{-0.9336\ A\ \hat{\imath} + 0.3584\ A\ \hat{\jmath}\} + \{0.4695\ B\ \hat{\imath} - 0.8829\ B\ \hat{\jmath}\} = 0$$

Vector equations contain as many scalar equations as there are dimensions to the problem. All the components that have an $\hat{\imath}$ must add to zero. Likewise, the same for $\hat{\jmath}$. Therefore, we get the following two equations: (I will drop the units of nautical miles for the time being.)

$$3.802 - 0.9336\ A + 0.4695\ B = 0$$

$$2.865 + 0.3584\ A - 0.8829\ B = 0$$

Now we just need to solve two equations with two unknowns. Take the first equation and solve for B.

$$B = \frac{3.802 - 0.9336\ A}{-0.4695} = -8.098 + 1.988\ A$$

Substitute this relationship for B into the second equation and solve for B.

$$2.865 + 0.3584\ A - 0.8829(-8.098 + 1.988\ A) = 0$$

$$2.865 + 0.3584\ A + 7.150 - 1.755\ A = 0$$

$$10.015 - 1.397\ A = 0$$

$$A = \frac{10.015}{1.397} = 7.169\ nmi$$

At the beginning of the problem, we set A as the distance between Noi and Poi. Therefore,

$$r_{n-p} = 7.17\ nmi$$

Converting this to kilometers gives

$$r_{n-p} = 7.169\ nmi\left(\frac{1.852\ km}{1.000\ nmi}\right) = 13.28\ km$$

Substituting the value of A into the modified first equation, we get

$$B = -8.098 + 1.988\ A = -8.098 + 1.988(7.169) = 6.154\ nmi$$

Therefore, the distance from Poi to Moi is

$$r_{p-m} = 6.15\ nmi$$

Or,

$$r_{p-m} = 6.154\ nmi\left(\frac{1.852\ km}{1.000\ nmi}\right) = 11.40\ km$$