Chapter 2 Problem 68 †

Solution

a) Find the cross product between $\vec{A} = 2.0\hat{\imath} - 4.0\hat{\jmath} + \hat{k}$ and $\vec{C} = 3.0\hat{\imath} + 4.0\hat{\jmath} + 10.0\hat{k}$.
First set up the matrix, with the unit vectors on the top row, components of vector A on the second row and the components of vector C on the third row. Find the determinant using expansion by minors.

$$\vec{A} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2.0 & -4.0 & 1.0 \\ 3.0 & 4.0 & 10.0 \end{vmatrix}$$

$$\vec{A} \times \vec{C} = \hat{\imath} \begin{vmatrix} -4.0 & 1.0 \\ 4.0 & 10.0 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 2.0 & 1.0 \\ 3.0 & 10.0 \end{vmatrix} + \hat{k} \begin{vmatrix} 2.0 & -4.0 \\ 3.0 & 4.0 \end{vmatrix}$$

Solving the determinant of the 2×2 matrices gives

$$\vec{A} \times \vec{C} = \hat{\imath}((-4.0)(10.0) - (4.0)(1.0)) - \hat{\jmath}((2.0)(10.0) - (3.0)(1.0)) + \hat{k}((2.0)(4.0) - (3.0)(-4.0))$$

$$\vec{A} \times \vec{C} = -44.0\hat{\imath} - 17.0\hat{\jmath} + 20.0\hat{k}$$

b) Find the cross product between $\vec{A} = 3.0\hat{\imath} + 4.0\hat{\jmath} + 10.0\hat{k}$ and $\vec{C} = 2.0\hat{\imath} - 4.0\hat{\jmath} + 1.0\hat{k}$.
Set up the matrix and find the determinant.

$$\vec{A} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3.0 & 4.0 & 10.0 \\ 2.0 & -4.0 & 1.0 \end{vmatrix}$$

$$\vec{A} \times \vec{C} = \hat{\imath} \begin{vmatrix} 4.0 & 10.0 \\ -4.0 & 1.0 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 3.0 & 10.0 \\ 2.0 & 1.0 \end{vmatrix} + \hat{k} \begin{vmatrix} 3.0 & 4.0 \\ 2.0 & -4.0 \end{vmatrix}$$

$$\vec{A} \times \vec{C} = \hat{\imath}((4.0)(1.0) - (-4.0)(10.0)) - \hat{\jmath}((3.0)(1.0) - (2.0)(10.0)) + \hat{k}((3.0)(-4.0) - (2.0)(4.0))$$

$$\vec{A} \times \vec{C} = 44.0\hat{\imath} + 17.0\hat{\jmath} - 20.0\hat{k}$$

Notice the sign has changed between parts a) and b) because we interchanged the order of the cross product. This results in a switching of two rows in the matrix. (Note: When swapping rows or columns in a matrix, the determinant maintains its magnitude, but changes its sign.)

c) Find the cross product between $\vec{A} = -3.0\hat{\imath} - 4.0\hat{\jmath}$ and $\vec{C} = -3.0\hat{\imath} + 4.0\hat{\jmath}$.
Set up the matrix and find the determinant using expansion by minors. If a coordinate is missing, there is an implied zero for that coordinate.

$$\vec{A} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -3.0 & -4.0 & 0 \\ -3.0 & 4.0 & 0 \end{vmatrix}$$

$$\vec{A} \times \vec{C} = \hat{\imath} \begin{vmatrix} -4.0 & 0 \\ 4.0 & 0 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} -3.0 & 0 \\ -3.0 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} -3.0 & -4.0 \\ -3.0 & 4.0 \end{vmatrix}$$

Solving the determinant of the 2×2 matrices gives

$$\vec{A} \times \vec{C} = \hat{\imath}((-4.0)(0) - (4.0)(0)) - \hat{\jmath}((-3.0)(0) - (-3.0)(0)) + \hat{k}((-3.0)(4.0) - (-3.0)(-4.0))$$

†Problem from University Physics by Ling, Sanny and Moebs (OpenStax)

$$\vec{A} \times \vec{C} = 0\hat{\imath} + 0\hat{\jmath} - 24.0\hat{k}$$
$$\vec{A} \times \vec{C} = -24.0\hat{k}$$

Notice that vectors A and C were in the x-y plane. The result of the cross product is perpendicular to the x-y plane and, therefore, only in the z-direction.

d) Find the cross product between $\vec{A} = -9.0\hat{\jmath}$ and $\vec{C} = -2.0\hat{\imath} + 3.0\hat{\jmath} + 2.0\hat{k}$.
Set up the matrix and find the determinant using expansion by minors.

$$\vec{A} \times \vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -9.0 & 0 \\ -2.0 & 3.0 & 2.0 \end{vmatrix}$$

$$\vec{A} \times \vec{C} = \hat{\imath}\begin{vmatrix} -9.0 & 0 \\ 3.0 & 2.0 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 0 & 0 \\ -2.0 & 2.0 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & -9.0 \\ -2.0 & 3.0 \end{vmatrix}$$

Solving the determinant of the 2×2 matrices gives

$$\vec{A} \times \vec{C} = \hat{\imath}((-9.0)(2.0) - (3.0)(0)) - \hat{\jmath}((0)(2.0) - (-2.0)(0)) + \hat{k}((0)(3.0) - (-2.0)(-9.0))$$

$$\vec{A} \times \vec{C} = -18.0\hat{\imath} + 0\hat{\jmath} - 18.0\hat{k}$$

Notice that vector A is only in the y-direction. Therefore, the cross product will be perpendicular to that direction and only have components in the x and z directions.