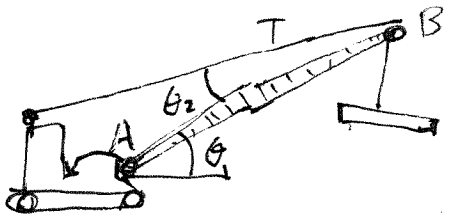
Chapter 12

Problem 42



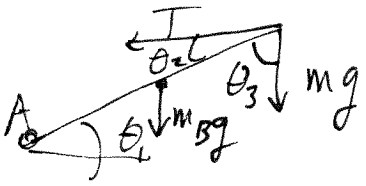
$\theta_1 = 30°$ $\theta_2 = 10°$

$L = 12.0\,m$ (length of Boom)

$m = 3000\,kg$ (load)

$m_B = 1800\,kg$ (mass of boom) (C.M. of Boom is at the geometric center)

Find Tension in cable, T



Place ~~pivot~~ pivot at A. Then

~~$\Sigma \vec{\tau} = -r_B F_B \sin\theta_3 - L mg$~~

$$\Sigma \vec{\tau} = \vec{\tau}_{\substack{\text{Boom}\\\text{weight}}} + \vec{\tau}_{load} + \vec{\tau}_{tension}$$

$$0 = -\frac{L}{2} m_B g \sin\theta_3 - L mg \sin\theta_3 + LT \sin\theta_2$$

$\theta_3 = 90 - \theta_1$
$= 90 - 30$
$= 60°$

Solve for T:

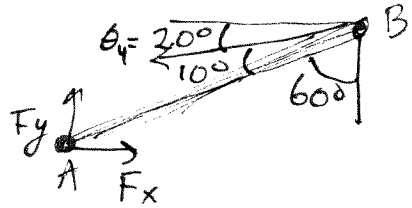
$$\frac{L}{2} m_B g \sin\theta_3 + Lmg \sin\theta_3 = LT \sin\theta_2$$

$$Lg \sin\theta_3 \left[\frac{m_B}{2} + m\right] = LT \sin\theta_2$$

$$T = g \frac{\sin\theta_3}{\sin\theta_2}\left[\frac{m_B}{2} + m\right] = (9.8\,m/s^2)\frac{\sin(60)}{\sin(10)}\left[\frac{1000}{2} + 3000\right]$$

$$\boxed{T = 171{,}000\,N}$$

Find Force at axle, A.



Use $\Sigma F = 0$

$$F_x \hat{i} + F_y \hat{j} - m_B g \hat{j} - mg\hat{j} - T\cos 20\,\hat{i} - T\sin 20\,\hat{j} = 0$$

$$\vec{F}_{axle} = F_x \hat{i} + F_y \hat{j} = T\cos 20\,\hat{i} + T\sin 20\,\hat{j} + m_B g\,\hat{j} + mg\,\hat{j}$$

$$= (171{,}000)\cos 20\,\hat{i} + [171{,}000 \sin 20 + 1000(9.8) + 3000(9.8)]\,\hat{j}$$

$$\boxed{\vec{F}_{axle} = \{161{,}000\,\hat{i} + 97{,}700\,\hat{j}\}\,N}$$

$$\boxed{\vec{F}_{axle} = 188{,}000\,N \angle 31.3°}$$

Notice: $\vec{F}_{axle}$ not parallel with Boom