Chapter 11

Problem 68

Geosynchronous Orbit $R_o = 4.21640 \times 10^7$ m

After collision, The orbit is elliptical.

When at apogee (furthest from earth) what is its speed? $R_f = R_a = 4.50000 \times 10^7$ m

Assume angular momentum is conserved.

Geosynchronous orbit → one orbit in 24 hours $T = 24\,hr\left(\frac{3600\,s}{1\,hr}\right) = 86{,}400\,s$

The initial speed is $v_o = \frac{d}{T} = \frac{2\pi r}{T}$

$$v_o = \frac{2\pi(4.21640 \times 10^7 m)}{(86{,}400 s)} = 3{,}066\ m/s = 3.066\ \frac{km}{s}$$

At These distances, The satellite is easily approximated as a point particle.

The moment of inertia is $I = MR^2$

$\omega = \frac{v}{R}$, so $\omega_o = \frac{v_o}{R_o}$ and $\omega_f = \frac{v_f}{R_f}$

$I_o = MR_o^2$ and $I_f = MR_f^2$

By conservation of angular ~~momentum~~ angular momentum

$$L_o = L_f$$

$$I_o\omega_o = I_f\omega_f$$

$$(MR_o^2)\left(\frac{v_o}{R_o}\right) = (MR_f^2)\left(\frac{v_f}{R_f}\right)$$

$$MR_o v_o = MR_f v_f$$

$$\therefore v_f = \frac{\cancel{M}R_o v_o}{\cancel{M}R_f} = \frac{(4.21640 \times 10^7 m)(3{,}066 m/s)}{(4.50000 \times 10^7 m)}$$

$$\boxed{v_f = 2{,}873\ m/s} = 2.873\ km/s$$