Chapter 11

Problem 53

$m = 2.0 \times 10^{30}\ kg$

$r_0 = 7.0 \times 10^5\ km = 7.0 \times 10^8\ m$

$T = 28\ days \left(\frac{24\ hr}{1\ day}\right)\left(\frac{3600\ s}{1\ hr}\right) = 2.42 \times 10^6\ s$

$r_f = 3.5 \times 10^3\ km = 3.5 \times 10^6\ m$

What would its period be if the sun collapsed to a white dwarf?

Moment of a solid sphere is $I = 2/5\, MR^2$

angular velocity is $\omega = \frac{\theta}{T}$

Brute force approach

$I_0 = \frac{2}{5}MR^2 = \frac{2}{5}(2 \times 10^{30}\ kg)(7.0 \times 10^8\ m)^2$

$I_0 = 3.92 \times 10^{47}\ kg \cdot m^2$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2.42 \times 10^6\ s}$

$\omega_0 = 2.60 \times 10^{-6}\ \frac{rad}{s}$

$I_f = \frac{2}{5}MR^2 = \frac{2}{5}(2.0 \times 10^{30}\ kg)(3.5 \times 10^6\ m)^2$

$I_f = 9.8 \times 10^{42}\ kg \cdot m^2$

Conservation of angular momentum

$L_0 = L_f$

$I_0 \omega_0 = I_f \omega_f$

$\omega_f = \frac{I_0 \omega_0}{I_f} = \frac{(3.92 \times 10^{47}\ kg m^2)(2.60 \times 10^{-6}\ \frac{rad}{s})}{(9.8 \times 10^{42}\ kg m^2)}$

$\omega_f = 0.104\ rad/s$

$\omega = \frac{\theta}{T} \rightarrow T_f = \frac{\theta}{\omega_f} = \frac{2\pi}{0.104\ \frac{rad}{s}}$

$\boxed{T_f = 60.4\ s}$

Elegant approach

$L_0 = L_f$

$I_0 \omega_0 = I_f \cdot \omega_f$

$\left(\frac{2}{5}Mr_0^2\right)\left(\frac{\theta}{T_0}\right) = \left(\frac{2}{5}Mr_f^2\right)\left(\frac{\theta}{T_f}\right)$

$\frac{2}{5}M\theta$ cancels on each side

Then $\frac{r_0^2}{T_0} = \frac{r_f^2}{T_f}$

and Then $T_f = \frac{r_f^2}{r_0^2}T_0 = \left(\frac{r_f}{r_0}\right)^2 T_0$

$T_f = \left[\frac{(3.5 \times 10^6\ m)}{(7.0 \times 10^8\ m)}\right]^2 (2.42 \times 10^6\ s)$

$T_f = \left[5.0 \times 10^{-3}\right]^2 (2.42 \times 10^6\ s)$

$\boxed{T_f = 60.5\ s}$