Chapter 11

Problem 42

$m = 1.0\,kg$

$\vec{r} = (2.0\hat{i} - 4.0\hat{j} + 6.0\hat{k})\,m$

$\vec{v} = (-1.0\hat{i} + 4.0\hat{j} + 1.0\hat{k})\,m/s$

$\vec{F} = (10.0\hat{i} + 15.0\hat{j})\,N$

a) What is the angular momentum about the origin?

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = m(\vec{r} \times \vec{v})$$

$$\vec{r} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 6 \\ -1 & 4 & 1 \end{vmatrix} = \hat{i}\begin{bmatrix} -4 & 6 \\ 4 & 1 \end{bmatrix} - \hat{j}\begin{bmatrix} 2 & 6 \\ -1 & 1 \end{bmatrix} + \hat{k}\begin{bmatrix} 2 & -4 \\ -1 & 4 \end{bmatrix}$$

$$= [(-4)(1) - (4)(6)]\hat{i} - [(2)(1) - (-1)(6)]\hat{j} + [(2)(4) - (-1)(-4)]\hat{k}$$

$$= (-4 - 24)\hat{i} - (2 + 6)\hat{j} + (8 - 4)\hat{k}$$

$$= (-28\hat{i} - 8\hat{j} + 4\hat{k})\,m^2/s$$

$$\vec{L} = m(\vec{r} \times \vec{v}) = (1.0\,kg)(-28\hat{i} - 8\hat{j} + 4\hat{k})\,\frac{m^2}{s}$$

$$\boxed{\vec{L} = (-28\hat{i} - 8\hat{j} + 4\hat{k})\,kgm^2/s}$$

b) What is the torque about the origin?

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 6 \\ 10 & 15 & 0 \end{vmatrix} = \begin{bmatrix} -4 & 6 \\ 15 & 0 \end{bmatrix}\hat{i} - \begin{bmatrix} 2 & 6 \\ 10 & 0 \end{bmatrix}\hat{j} + \begin{bmatrix} 2 & -4 \\ 10 & 15 \end{bmatrix}\hat{k}$$

$$= [(-4)(0) - (15)(6)]\hat{i} - [(2)(0) - (10)(6)]\hat{j} + [(2)(15) - (10)(-4)]\hat{k}$$

$$\boxed{\vec{\tau} = (-90\hat{i} + 60\hat{j} + 70\hat{k})\,N \cdot m}$$

c) What is the ~~angular~~ time rate of change of the particle's angular momentum at this instant?

$$\vec{\tau} = \boxed{\frac{d\vec{L}}{dt} = [-90\hat{i} + 60\hat{j} + 70\hat{k}]\,N \cdot m}$$