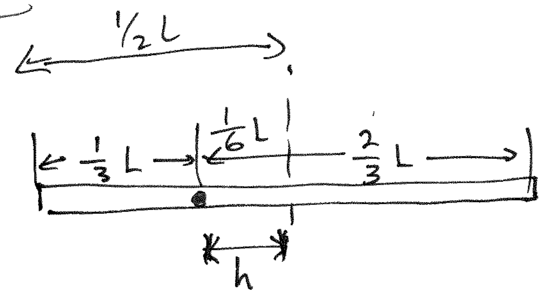
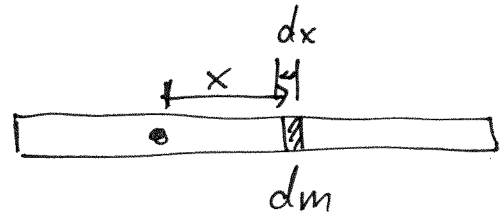
Chapter 10 Problem 70

Find the moment of inertia for the rod by integration.





for an infinitesimal slice of rod, ~~dm~~ dx, it has a mass of dm. Assuming the rod has uniform cross-section and composition, then dm is related to dx by a constant λ.

If the full length of the rod is L and the total mass is M, then

$$M = \lambda \cdot L \longrightarrow \lambda = \frac{M}{L}$$

and $dm = \lambda \cdot dx$

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now find the moment of inertia

$$I = \int r^2 dm = \int_{-L/3}^{\frac{2L}{3}} x^2 \cdot \lambda\, dx = \lambda \int_{-L/3}^{\frac{2L}{3}} x^2 dx$$

For a rod

$$\boxed{I_{cm} = \frac{1}{12} mL^2}$$

$$= \frac{\lambda}{3} x^3 \Big|_{-L/3}^{2L/3} = \frac{\lambda}{3}\left[\left(\frac{2}{3}L\right)^3 - \left(-\frac{L}{3}\right)^3\right]$$

$$= \frac{\lambda}{3}\left[\frac{8}{27}L^3 + \frac{1}{27}L^3\right] = \frac{\lambda}{3} \cdot \frac{9}{27} L^3$$

since $\lambda = \frac{M}{L}$ then $I = \frac{M}{L} \cdot \frac{1}{3} \frac{9}{27} L^3 = \boxed{\frac{1}{9} ML^2}$

Parallel axis Theorem

$$I = I_{cm} + mh^2 = \frac{1}{12} mL^2 + m\left(\frac{L}{6}\right)^2 = \frac{1}{12} mL^2 + \frac{1}{36} mL^2$$

$$= \frac{3}{36} mL^2 + \frac{1}{36} mL^2 = \frac{4}{36} mL^2 = \boxed{\frac{1}{9} mL^2}$$ The same