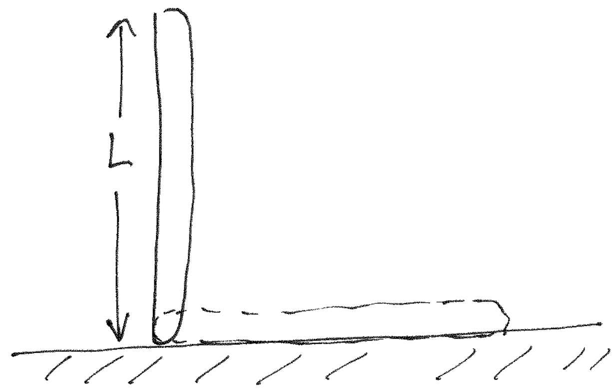
Chapter 10 Problem 103



The end of the rod doesn't slip.
What is the linear velocity of the upper end when it hits the floor.

Assuming no work is done by friction as it falls, conservation of energy can be used. This is a good assumption since the end does not slip.

Treating the lower end as a pivot point, we can calculate the moment of inertia of the rod.

$$I = \frac{1}{3}ML^2$$

Final Kinetic energy is $K_f = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{M}{3}L^2\right)\omega^2$

$$K_f = \frac{1}{6}ML^2\omega^2$$

Initial Potential energy is based on how high the center of mass is above the floor. Therefore

$$U_0 = mgh = mg\left(\frac{L}{2}\right) = \frac{1}{2}MgL$$

Now use conservation of energy

$$U_0 + K_0 = U_f + K_f$$

$$\frac{1}{2}MgL + 0 = 0 + \frac{1}{6}ML^2\omega^2 \rightarrow \frac{1}{2}MgL = \frac{1}{6}ML^2\omega^2$$

But tangential velocity at the upper end is $v_t = L\omega$

$$\therefore \frac{1}{2}MgL = \frac{1}{6}Mv_t^2 \rightarrow \frac{6}{M}\frac{1}{2}MgL = v_t^2 \rightarrow \boxed{v_t = \sqrt{3gL}}$$