Chapter 9 Problem 44 [†]

Given

$m_c = 950\ kg$
$\vec{v}_c = \{32\hat{i} + 17\hat{j}\}\ m/s$
$m_w = 450\ kg$
$\vec{v}_w = \{12\hat{i} + 14\hat{j}\}\ m/s$

Solution

If the car and wagon stick together, find their velocity after the collision.

By conservation of momentum, the momentum of the car-wagon system before and after the collision is the same.

$$\vec{p}_{before} = \vec{p}_{after}$$

$$\vec{p}_c + \vec{p}_w = \vec{p}_{cw}$$

$$m_c\vec{v}_c + m_w\vec{v}_w = (m_c + m_w)\vec{v}_{cw}$$

$$\vec{v}_{cw} = \frac{m_c\vec{v}_c + m_w\vec{v}_w}{(m_c + m_w)}$$

$$\vec{v}_{cw} = \frac{(950\ kg)\{32\hat{i} + 17\hat{j}\}\ m/s + (450\ kg)\{12\hat{i} + 14\hat{j}\}\ m/s}{(950\ kg + 450\ kg)}$$

$$\vec{v}_{cw} = \frac{\{30400\hat{i} + 16150\hat{j}\}\ kg \cdot m/s + \{5400\hat{i} + 6300\hat{j}\}\ kg \cdot m/s}{(1400\ kg)}$$

$$\vec{v}_{cw} = \frac{\{35800\hat{i} + 22450\hat{j}\}\ kg \cdot m/s}{(1400\ kg)}$$

$$\vec{v}_{cw} = \{25.6\hat{i} + 16.0\hat{j}\}\ m/s$$

In polar coordinates this velocity is

$$v = \sqrt{(25.6\ m/s)^2 + (16.0\ m/s)^2} = 30.2\ m/s$$

$$\theta = \tan^{-1}\left(\frac{16.0\ m/s}{25.6\ m/s}\right) = 32.0^\circ$$

[†]Problem from Essential University Physics, Wolfson