Chapter 6 Problem 53 [†]

Given
$F = a\sqrt{x} = ax^{1/2}$
$a = 9.5\ N/m^{1/2}$

Solution
a) Find the work done by this force from $x = 0\ m$ to $x = 3.0\ m$.

Since the force is changing with position, we use the following definition of work.

$$W = \int F dx$$

Integrate the force between x_1 and x_2 gives

$$W_{x_2-x_1} = \int_{x_1}^{x_2} ax^{1/2} dx = \tfrac{2}{3} a x^{3/2}\big|_{x_1}^{x_2} = \tfrac{2}{3} a \left(x_2^{3/2} - x_1^{3/2} \right) \tag{1}$$

Now substitute in $x_1 = 0\ m$ and $x_2 = 3.0\ m$ and $a = 9.5\ N/m^{1/2}$

$$W = \tfrac{2}{3}(9.5\ N/m^{1/2}) \left((3.0\ m)^{3/2} - (0\ m)^{3/2} \right)$$

$$W = 32.9\ J$$

b) Find the work done by this force from $3.0\ m$ to $6.0\ m$.

Equation (1) can be used with the limits of $x_1 = 3.0\ m$ and $x_2 = 6.0\ m$.

$$W = \tfrac{2}{3}(9.5\ N/m^{1/2}) \left((6.0\ m)^{3/2} - (3.0\ m)^{3/2} \right)$$

$$W = 60.2\ J$$

c) Find the work done by this force from $6.0\ m$ to $9.0\ m$.

Equation (1) can be used with the limits of $x_1 = 6.0\ m$ and $x_2 = 9.0\ m$.

$$W = \tfrac{2}{3}(9.5\ N/m^{1/2}) \left((9.0\ m)^{3/2} - (6.0\ m)^{3/2} \right)$$

$$W = 77.9\ J$$

[†]Problem from Essential University Physics, Wolfson