

Given $m = 3700 \ kg$ $T_1 = T_2 = 1100 \ N$ $\theta_1 = \theta_2 = 25^{\circ}$

Solution

a) Find the force of the water if the velocity is constant.

Drawing the free-body diagram, we get something similar to the diagram above. Choose the coordinate system so that the x-coordinate is to the right. Newton's 2^{nd} law gives the equation

$$\Sigma \vec{F} = m\vec{a}$$

$$\vec{T}_1 + \vec{T}_2 + \vec{F}_w = m\vec{a}$$
 (1)

Write out each of the forces in unit vector notation.

$$T_{1}\cos\theta_{1}\hat{i} + T_{1}\sin\theta_{1}\hat{j} + T_{2}\cos\theta_{2}\hat{i} - T_{2}\sin\theta_{2}\hat{j} - F_{w}\hat{i} = 0$$

Since the angles are the same and the tensions are the same we can drop the subscripts on these values.

$$T\cos\theta\hat{i} + T\sin\theta\hat{j} + T\cos\theta\hat{i} - T\sin\theta\hat{j} - F_w\hat{i} = 0$$

Notice that the acceleration term is set to zero because the barge is moving at constant velocity. The x-component equation is

$$T\cos\theta + T\cos\theta - F_w = 0\tag{2}$$

and the y-component equation is

$$T\sin\theta - T\sin\theta = 0\tag{3}$$

Use equation (2) to solve for the force of the water.

$$F_w = T\cos\theta + T\cos\theta = 2T\cos\theta = 2(1100 N)\cos(25^\circ)$$
$$F_w = 1990 N$$

b) Find the force of the water if the acceleration is 0.16 m/s^2 .

The free-body diagram still applies and all the work up to equation (1). When the forces are written out in unit vector notation, the acceleration is in the positive x-direction. This leads to the equation

$$T_1 \cos \theta_1 \hat{i} + T_1 \sin \theta_1 \hat{j} + T_2 \cos \theta_2 \hat{i} - T_2 \sin \theta_2 \hat{j} - F_w \hat{i} = ma\hat{i}$$

[†]Problem from Essential University Physics, Wolfson

Since the tensions and angles are the same, we get

$$T\cos\theta\hat{i} + T\sin\theta\hat{j} + T\cos\theta\hat{i} - T\sin\theta\hat{j} - F_w\hat{i} = ma\hat{i}$$

The x-component equation is

$$T\cos\theta + T\cos\theta - F_w = ma \tag{4}$$

and the y-component equation is

$$T\sin\theta - T\sin\theta = 0\tag{5}$$

Use equation (4) to solve for the force of the water.

$$F_w = T \cos \theta + T \cos \theta - ma = 2T \cos \theta - ma$$

 $F_w = 2(1100 \ N) \cos(25^\circ) - (3700 \ kg)(0.16 \ m/s^2)$
 $F_w = 1400 \ N$