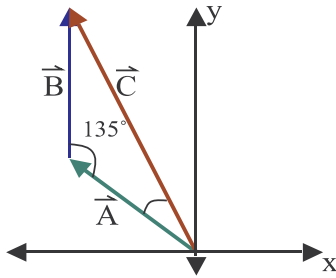


Chapter 3 Problem 19 †



Given

$$\vec{A} = 360 \text{ km } \angle 135^\circ$$

$$\vec{B} = 400 \text{ km } \angle 90^\circ$$

Solution

Find the displacement from the start.

Solving this graphically you get the diagram given above. We can now use the law of cosines to find the magnitude of vector C.

$$C^2 = A^2 + B^2 - 2AB \cos \gamma$$

$$C^2 = (360 \text{ km})^2 + (400 \text{ km})^2 - 2(360 \text{ km})(400 \text{ km}) \cos 135^\circ$$

$$C^2 = 493,247 \text{ km}^2$$

$$C = 702 \text{ km}$$

Next find the angle *beta* using the law of sines.

$$\frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$$

$$\beta = \sin^{-1} \left(\frac{B \sin \gamma}{C} \right)$$

$$\beta = \sin^{-1} \left(\frac{400 \text{ km} \sin 135^\circ}{702 \text{ km}} \right) = 23.8^\circ$$

Since the direction of \vec{C} is an angle β less than the angle of \vec{A} , the direction of C is

$$\angle C = 111.2^\circ$$

The problem can also be solved using unit vectors.

First convert into unit vector notation.

$$\vec{A} = \left\{ A \cos \theta \hat{i} + A \sin \theta \hat{j} \right\} = \left\{ 360 \cos 135^\circ \hat{i} + 360 \sin 135^\circ \hat{j} \right\} \text{ km}$$

$$\vec{A} = \left\{ -254.6 \hat{i} + 254.6 \hat{j} \right\} \text{ km}$$

and

$$\vec{B} = \left\{ B \cos \theta \hat{i} + B \sin \theta \hat{j} \right\} = \left\{ 400 \cos 90^\circ \hat{i} + 400 \sin 90^\circ \hat{j} \right\} \text{ km}$$

†Problem from Essential University Physics, Wolfson

$$\vec{B} = \{400\hat{j}\} \text{ km}$$

Add the two vectors together to get the displacement.

$$\vec{C} = \vec{A} + \vec{B} = \{-254.6\hat{i} + 254.6\hat{j}\} \text{ km} + \{400\hat{j}\} \text{ km}$$

$$\vec{C} = \{-254.6\hat{i} + 654.6\hat{j}\} \text{ km}$$

Convert into polar coordinate notation.

$$C = \sqrt{(-254.6 \text{ km})^2 + (654.6 \text{ km})^2}$$

$$C = 702 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{654.6 \text{ km}}{-254.6 \text{ km}}\right) = -68.7^\circ$$

However, our answer should be in the 2nd quadrant not the first; therefore, add 180° to give

$$\vec{C} = 702 \text{ km } \angle 111.3^\circ$$