Chapter 15 Problem 53 [†]

Given

$$\rho_{helium} = 0.18 \ kg/m^3$$
$$m = 280 \ kg$$

Solution

a) Find the volume of helium needed to lift the balloon, the basket and its occupants.

Assuming the balloon is just floating in place, Newton's 2^{nd} law gives us

$$B - (m_{helium} + m)q = 0$$

where B is the buoyant force and gravity is acting on the mass of the helium and the mass of the balloon. The buoyant force is the mass of the displaced air times gravity.

$$m_{air}g - (m_{helium} + m)g = 0$$

$$m_{air} - m_{helium} - m = 0$$

The mass of the displaced air is the density of air times the volume of the balloon and the mass of the helium is the density of helium times the volume of the balloon.

$$\rho_{air}V - \rho_{helium}V - m = 0$$

Solving for volume gives

$$(\rho_{air} - \rho_{helium})V = m$$

$$V = \frac{m}{(\rho_{air} - \rho_{helium})}$$

$$V = \frac{(280 \ kg)}{(1.2 \ kg/m^3 - 0.18 \ kg/m^3)} = 275 \ m^3$$
(1)

If any additional lift is needed, we would need to have more than this volume of helium.

b) Find the volume if hot air is used.

Equation 1 from part a can be used where the density of helium is replaced by the density of hot air.

$$V = \frac{m}{(\rho_{air} - \rho_{hot})}$$

The density of hot air is 10% less than air therefore

$$\rho_{hot} = (1 - .1)\rho_{air} = 0.9(1.2 \ kg/m^3) = 1.08 \ kg/m^3$$

$$V = \frac{(280 \ kg)}{(1.2 \ kg/m^3 - 1.08 \ kg/m^3)} = 2330 \ m^3$$

[†]Problem from Essential University Physics, Wolfson