Introduction to Astronomy	**Laboratory Exercise #6**
Name ____________________________	Date _______________
Partners __________________________	Section _____________

Size of the Moon

Purpose:
To calculate the radius and mass of the moon and to understand the train of reasoning associated with our understanding of astronomical data.

Equipment:
Calculator Ruler Metal Cylinder Protractor

In this lab we will determine the mass of the moon by two different methods. Each of these methods reflect the way data is collected concerning the cosmos.

Trigonometry
Some times it is not practical or possible to directly measure the length of an object. Geometry can come to our aide when we consider that a triangle can be completely described by knowing the length of one side and two angles. Trigonometry standardizes this idea by defining three functions related to a right triangle.

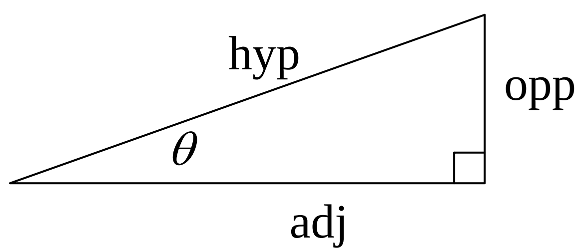


Figure 5 illustrates a right triangle where the lower left corner represents a 90° angle. If you know the angle, θ, or want to determine the angle, θ, you can label the sides of the triangle as illustrated in figure 6. The side opposite of the 90° angle is called the hypotenuse and is abbreviated as *hyp*. The side adjacent to the angle, θ, is abbreviated *adj* and the opposite side is abbreviated *opp*.

The three functions related to these quantities are defined as follows

$$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$$

If you want to determine the length of side opp, you can accurately measure the angle, q, and the side adj. Using the tangent function and a little algebra gives you

$$opp = adj \cdot \tan\theta$$

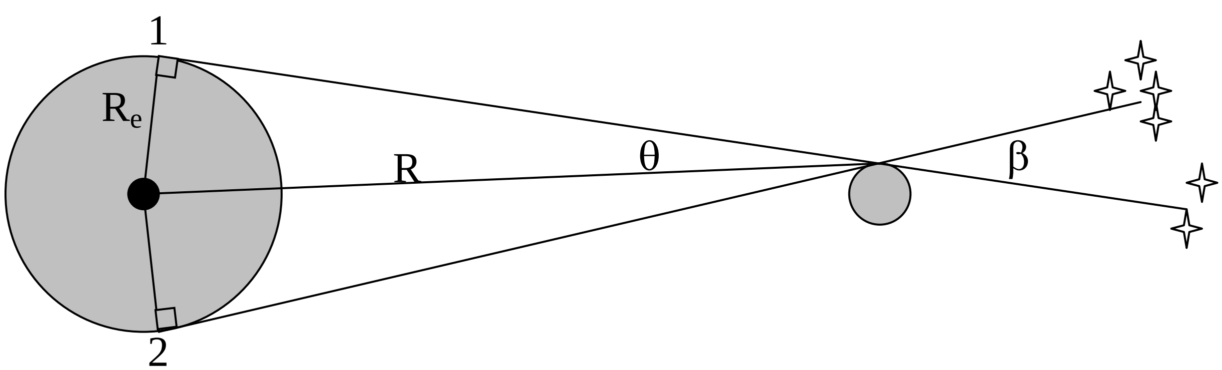
Parallax

You can perceive the distance of an object by its motion when you are moving. The most distant objects will appear to be stationary while close objects will appear to move more rapidly with their proximity to you. In this lab we will use this effect to measure the distance to the moon.

The position of a star in the sky is recorded as a pair of two angles. The first angle is called Right Ascension, RA, and is measured in units of hours, minutes and seconds. RA roughly goes from west to east and ranges from 0 to 24 hours. Only about 12 hours RA can be seen of the sky at any one time. The second angle measure is called Declination, DEC, and is measured in degrees, minutes and seconds. DEC roughly goes from south to north and ranges from –90° to +90°. The location of 0° in the sky changes as you travel north or south.

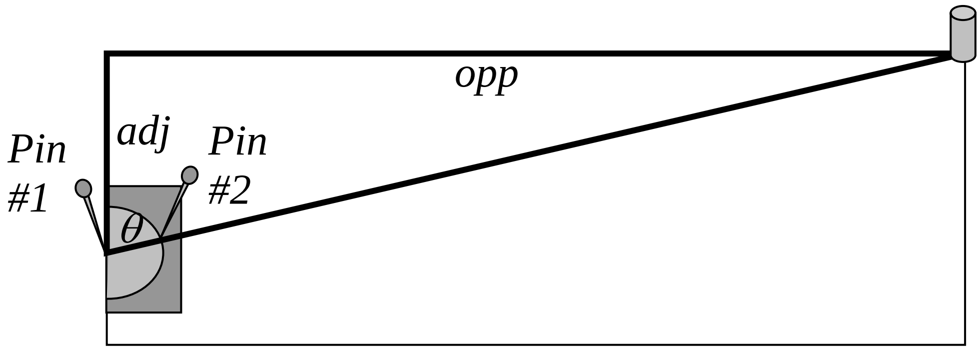
As the earth rotates about its axis, the position of the stars relative to each other remains fixed from an observer's point of view because of their great distance. However, the position of the moon relative to the stars does shift and the angle of shift is called parallax, β.

As illustrated below, two observers compare their measurement of the moon's location in the sky at the exact same time. Notice that two right triangles can be constructed. The corners of the triangle correspond to the observer's position, the center of the earth and the edge of the moon. The angle, θ, is equal to half the parallax angle, β. Since the radius of the earth, R_e, is know, the distance from the center of the earth to the moon, R, can be calculated.



TRIGONOMETRY

Place the protractor on a piece of wood and line it up with the edge of the table. Push a pin into the center back of the protractor. Place an object at the far end of the table as illustrated below. Stoop down so you can sight from the pin to the object at the end of the table. Take a second pin and place it on the curved edge of the protractor in such a way that the pins and the object line up. Carefully push the second pin into the wood.



Measure the distance from the first pin to the edge of the table, *adj*, and record that value below. Also record the angle, θ, from the edge of the table to the second pin.

adj = _______________ θ = ___________________

Calculate the length of the table using the tangent function and record the value below.

opp = _______________

Compare this result with your direct measurement made in part I of the lab. Assuming the direct measurement is very accurate, the percent error can be calculated with the following formula. Record the percent error below. (You do not need to include an uncertainty on calculations of percent error.)

$$\%error = \frac{opp - length}{length} \times 100\% = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Mass from the Moon's Size

This first method uses parallax to determine the distance to the moon. Once the moon's distance is known and its size, we will assume an average density for the moon.

1. Distance to the moon from parallax.

Take attachments number 1 – 3 to do this portion of the lab. Attachments one and two are views of the sky on April 16, 2002 at 8:35 AM GMT from two different spots on the earth. The one is where the moon is just setting and the other is where the moon is just rising. If you look at the two views, you will notice that the position of the moon is different with respect to the background stars. This difference in position is due to 'parallax'.

From the earth-moon diagram on the previous page you can see how parallax arises from the two views of the moon. When the moon is setting (1) the moon appears to be amongst one cluster of stars, while when it is rising (2) it appears to be amongst a different cluster of stars. Since we measure all star locations as angles, the parallax for the moon's position will be just the angular distance between the two locations of the moon.

Once the parallax angle is known, a right trangle can be constructed with one leg of the triangle being the radius of the earth and the opposite angle, θ, being half of the parallax angle, β.

Calculation:

a) Plot the moon's location from star chart 1 on to star chart 2.

b) Measure the distance between Aldebaran and the central star of the Pleiades (Alcyone) on the star chart.

d_{aa} = ________________________ cm

c) Measure the distance between the two positions of the moon on the star chart and record it below.

d_{moon} = ______________________cm

d) Using the third attachment, which is a star chart for the constellation Taurus, measure the distance between Aldebaran and Alcyone.

d_{sc} = ______________________cm

e) On this same star chart there are markings for declenation and right ascention, the coordinates for stars on the Celestial Sphere. Measure the distance between RA = 3h DEC 0° and RA = 3h DEC +10°. This measurement corresponds to 10° in the sky.

d_{10} = ______________________cm

f) Using these measurement determine the parallax angle for the moon. Notice this is just a process of applying conversion factors. We want to convert dmoon into an angle and we know that $d_{aa} = d_{sc}$ and $d_{10} = 10°$.

$$\beta = d_{moon}\left(\frac{d_{sc}}{d_{aa}}\right)\left(\frac{10°}{d_{10}}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ °$$

g) From parallax determine the angle opposite of the earth's radius, θ.

θ = ________________________°

h) From trigonometry the ratio of opposite side to hypotenuse is the sine function. Using the diagram for the earth and moon the sine of θ becomes the ratio of the earth's radius, R_e, to the earth-moon distance, R.

$$\sin\theta = \frac{opp}{hyp} = \frac{R_e}{R}$$

The radius of the earth is $R_e = 6.38 \times 10^6$ m

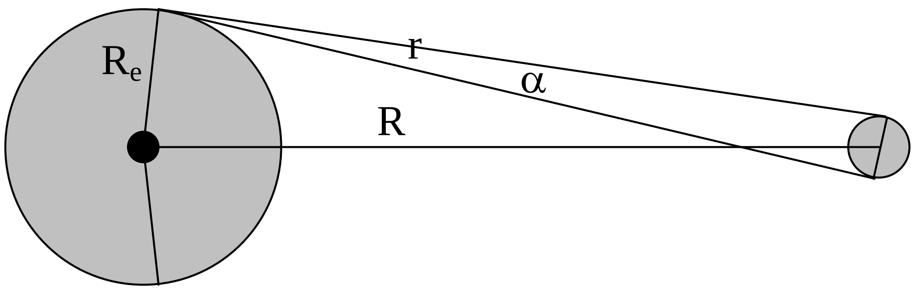
$$R = \frac{R_e}{\sin\theta} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{m}$$

i) Determine the diameter of the moon by using measuring its size on the star chart.

w_{moon} = ______________________cm

j) Convert this distance into an angle as we did before.

$$\alpha_{moon} = w_{moon}\left(\frac{d_{sc}}{d_{aa}}\right)\left(\frac{10°}{d_{10}}\right) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ °$$



k) The distance between the observer and the moon is r, which is the adjacent side of the triangle. Again using trigonometry we have

$$\cos\theta = \frac{adj}{hyp} = \frac{r}{R}$$

The observer distance is then

$$r = R\cos\theta = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{m}$$

l) The diameter of the moon can now be determined by using the resolution formula from class. Remember, α needs to be in terms of seconds for this formula to work.

$$s = \frac{ra}{2.06\times10^5} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{m}$$

m) Once you have the diameter of the moon the radius and volume can be calculated.

r_{moon} = s/2 = __________________________m

$$V = \frac{4}{3}\pi\, r_{moon}{}^3 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \text{m}^3$$

n) Assuming the average density of the moon is the same as the earth's, calculate the mass of the moon. The average density of the earth is 5520 kg/m^3.

m = ρV = _______________________kg

Mass from a satellite's orbit of the moon

On July 22, 1967 a satellite was placed in orbit around the moon. This satellite was named Explorer 35 and the attached data gives the position of Explorer 35 at 15 minute intervals. From this data plot the orbit of the satellite and determine the orbital period and distance of the semi-major axis.

a = ____________________Lunar Radii

P = ____________________hours

Convert the semi-major axis into meters and the period into seconds

(1 lunar radius = 1.738×10^6m) (1 hour = 3600 s)

a = ____________________m

P = ____________________s

Combining Kepler's Laws with Newton's form of gravitational force we get

$$P^2 = \frac{4\pi^2}{GM} a^3$$

Solving for M gives (G = 6.67×10^{-11} Nkg^2/m^2)

$$M = \frac{4\pi^2}{GP^2} a^3 = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ kg}$$

Questions:

1. How do the two masses compare? Are they close?

2. Which measurement do you think is most accurate?

3. What are the assumptions that go into each of these calculations?